Int J Theor Phys (2008) 47: 2259–2267
DOI 10.1007/s10773-008-9658-5

Four-Dimensional Newton's Constant in Brane World with a Sloping Extra Dimension

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Received: 22 October 2007 / Accepted: 8 January 2008 / Published online: 1 February 2008

Abstract We show the four-dimensional Newton's constant obtained naturally from five-dimensional brane world with a tinily sloping extra dimension, which is independent of the bulk Weyl tensor. The corresponding universe is stiff fluid dominated when the slope of extra dimension is very small. Otherwise, the universe may be undergoing a self-acceleration at present epoch and have a decelerated phase in very recent past.

1 Introduction

The brane-world idea, according to which the observable four-dimensional universe is a hypersurface embedded in a higher-dimensional space-time, has attracted many attention in recent years [1–3]. One merit of this idea is that four-dimensional objects should have a higher-dimensional origin. It has been understood as a route towards reconciling some hierarchy problems. The prominent suggestion has been to lower the fundamental scale of gravity all the way to the weak scale by introducing large [4, 5] or warp extra dimensions [6]. And the small four-dimensional cosmological constant maybe hinge on an unknown fine tuning between the five-dimensional one on bulk and brane [6, 7]. However, if the five-dimensional cosmological constant does not enter on the brane, the (four-dimensional) Newton's constant does not too [8, 9]. Whilst the Friedmann law on the brane depends quadratically on the brane energy density, up to a dark radiation term due to the bulk Weyl tensor, rather than linearly as in four-dimensional cosmology [10, 11]. This would contradict the principle that

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physics should appear four-dimensional at low energy. It has been suggested by Csáki et al. [12] that the unconventional forms of the Friedmann law is a consequence of requiring the radion, i.e. the distance between the branes, to be static without the addition of a stabilizing potential. Recently, Khoury and Zhang [13] has pointed out that the four-dimensional Friedmann law should hold at low energy, independent of whether the radion evolves with time or not. Using an approximate solution to the five-dimensional equations of motion, they show that the linear term may originate from the projection of the bulk Weyl tensor onto the brane instead as described as a new form of dark energy. The Newton's constant is determined by radion field but not its time derivative indeed. However, one should notice that their approximate solution is restricted in some special Weyl tensor. Assume the metric

$$ds^2 = g_{ab}dx^a dx^b + g_{55}dy^2,$$

and note that the unit vector normal to the $y = 0$ brane is $n^\alpha = [0, 0, 0, 0, (g_{55})^{-\frac{1}{2}}]$. The electric part of five-dimensional Weyl tensor can be written in detail

$$\begin{aligned} E_{ab} = O(\tau^2) - \frac{1}{2} g^{55} \widehat{\partial_5^2 g_{ab}} + \frac{1}{4} g^{55} g^{55} \partial_5 g_{55} \partial_5 g_{ab} \\ - \frac{1}{2} g^{55} \left(g_{ac} \partial_b g^{cd} \partial_d g_{55} + \partial_b \partial_a g_{55} - \frac{1}{2} g^{55} \partial_b g_{55} \partial_a g_{55} + g_{ac} g^{de} \partial_e g_{55} \Gamma^c_{db} \right). \end{aligned} \quad (1)$$

Besides $O(\tau^2)$, which denotes the higher order terms of energy-momentum tensor τ_{ab} on brane under Israel junction condition [14], one can find that E_{ab} consists of the term with non-distributional part of $\partial_5^2 g_{ab}$, denoted as $\widehat{\partial_5^2 g_{ab}}$ [11], and the terms with derivative of g_{55}. Generally, they all contain the gravitational effects of the bulk and may provide the linear density with corresponding Newton's constant, which should be constituted by the undetermined bulk ingredients $\widehat{\partial_5^2 g_{ab}}$, g_{55} and its derivative, noting $\partial_5 g_{ab}$ is determined by the energy-momentum tensor under Israel junction condition. If we adopt the approximate solution of two branes model as Ref. [13] (see (13)), only the second term $-\frac{1}{2} g^{55} \widehat{\partial_5^2 g_{ab}}$. (For example, its 00 component is $-\frac{1}{6d_0} \sum_{i=1}^{2} (2\rho_i + 3p_i)$.) provides a non-vanishing term with linear density, with the Newton's constant determined by the radion field, independent with its time derivative. However, for example, if the third term appears, the grads of radion along the extra dimension may compensate the Newton's constant, even when we consider a spatial flat universe and impose that the Newton's constant should be independent of whether the radion evolves with time or not, which only results that the second row of (1) disappears.

It is clear that the linear term in Friedmann law is still absent when $\widehat{\partial_5^2 g_{ab}}$ is trivial and the radion is static or constant [15] like the bulk with the fixed brane, or when the Weyl tensor is trivial for a five-dimensional AdS bulk, which, as commented by Khoury and Zhang themselves, implies that the energy density on the branes cancel each other to leading order. So in general case, what is the higher-dimensional origin of Newton's constant? It is well known that bulk is usually split into a direct product of brane and extra dimension, in other words, the extra dimension is normal to the brane. We will show that the tiny departure i.e., the perturbation of the normal extra dimension, may naturally result a conventional Newton's constant.

The central question of cosmological perturbations is to consider the unconventional cosmological effects induced by the un-decouple internal and external fluctuations to test the brane-world idea. In the pioneer work [16, 17] of de Bruck et al., the evolution of cosmological scale and then tensor perturbations are discussed. Usually, the five-dimensional analysis is rendered simpler by choosing a gauge in which the cross metrics between the brane and extra dimension vanish, even in recent attempting to go beyond the limiting low-energy approximation [18–20]. However, in one of our previous work [21], we studied the relationship between two transversal submanifolds and global manifold, and found that the induced cross metrics play essential roles to bridge the two transversal submanifolds. We consider the presence of induced cross metrics is reasonable because of two facts: (i) Though a manifold can be flatted locally, it can not be globally, i.e. the cross metrics do exist globally and generally. (ii) Even the manifold is flatted locally by relinearing the coordinates, the physical meaning of the new coordinates is not very clear. So we hope to work still in the original coordinate frame. In this paper, we will care about the presence of these cross metrics and still work to low-energy approximation. This is motivated by a naive intuition that the slope of extra dimension changing the geometrical structure of bulk should give unconventional results compared by the other perturbations. We expect that the unconventionality may be relative to the unclear physical reason for the acceleration of present universe with decelerated phase in recent past, which inferred in high redshift surveys of type Ia supernovae [22–26], and independently implied from observations of the cosmic microwave background (CMB) by the WMAP satellite [27] and other CMB experiments [28, 29].

In the rest of paper, we begin by obtaining the general formalism of four-dimensional effective Einstein equation in five-dimensional brane world with a tinily sloping extra dimension. Then we look for the conventional Friedmann law on the spatially flat brane with an extra dimension transversal to time for cosmological applications. At last an approximate solution of five-dimensional Einstein equation on whole space-time is obtained and corresponding results about expansion behavior of universe are presented.

2 Four-Dimensional Einstein Equation

Consider our 3-brane world embedded in five-dimensional bulk space-time, with one-dimensional sloping extra dimension. To investigate the new origin of Newton's constant, we assume the radion field or the metric g_{55} is static. It is then possible to determine the explicit dependence of the metric on extra dimension. The restriction allows us to go to the gauge $g_{55} = 1$ [15]. We select a coordinate y such that the hypersurface $y = 0$ coincides with the brane world. Under these assumptions, we can write the five-dimensional metric in more explicit form

$$ds^2 = g_{ab}dx^a dx^b + g_{a5}dx^a dy + dy^2,$$

where the cross metric g_{a5} is first-order small and all the higher-order small quantities will be omitted.

One can find that four-dimensional Einstein tensor can be derived from the five-dimensional one

$$^{(4)}G_{ab} = {}^{(5)}G_{ab} + \theta_{ab} + \omega_{ab},$$

$$\theta_{ab} = -2K_{ac}K_b^c + K_{ab}K - \frac{1}{2}g_{ab}(K^2 - 3K_{cd}K^{cd}) + \partial_5 K_{ab} - g_{ab}g^{cd}\partial_5 K_{cd}$$

where $K_{ab} = \frac{1}{2}\partial_5 g_{ab}$, that is just the extrinsic curvature in Gauss normal coordinate. Introducing the Weyl tensor and considering the 55 component of five-dimensional Einstein tensor, θ_{ab} can be recovered as the familiar difference between two Einstein tensors on the brane [8, 30]. The additional difference ω_{ab} is the terms with cross metric. It is expiatory and we do not write it clearly. However we must point out that there is no double extra dimensional derivative at leading order. Thus, one can assume the energy-momentum tensor in an empty bulk. (In this paper, the five-dimensional Planck mass is unit.)

$$T_{\mu\nu} = \begin{bmatrix} \tau_{ab}\delta(y) & \tau_{a5}\delta(y) \\ \tau_{5a}\delta(y) & 0 \end{bmatrix}$$

and obtain the junction condition

$$[K_{ab}] = -\left(\tau_{ab} - \frac{1}{3}g_{ab}\tau_{ab}\right)$$

similar to Israel junction condition. Note the τ_{a5} should be self-consistent with the junction condition. Further, one still can impose the Z_2 symmetry, at least in the neighborhood of brane. In fact, the only difference with the direct-product case is that it is the symmetry along a slightly sloping extra dimension rather a normal one. Then we have

$$K_{ab} = -\frac{1}{2}\left(\tau_{ab} - \frac{1}{3}g_{ab}\tau\right). \tag{2}$$

Noticing that we work at the low-energy case, all the higher-order terms $O(\tau^2)$ are omitted,

$$\theta_{ab} = \widehat{\partial_5 K_{ab}} - g_{ab}g^{cd}\widehat{\partial_5 K_{cd}} + O(\tau^2). \tag{3}$$

Moreover, to investigate the new origin of Newton's constant, we only care about the case with vanishing $\widehat{\partial_5 K_{ab}}$. Then the four-dimensional Einstein equation is simplified as

$$^{(4)}G_{ab} = \omega_{ab} + O(\tau^2). \tag{4}$$

Since K_{ab} is determined by the energy-momentum tensor equation (2), one can find that the undetermined bulk ingredients in ω_{ab} are only the cross metric and its derivative. They should provide a new origin of Newton's constant, independent with the previous non-distributional part of the double extra dimensional derivative of four-dimensional induced metric and the radion field in (1). This is one of the essential result of this paper and will be seen more explicitly in Friedmann equation below.

3 Friedmann Equation

We now derive the Friedmann equation using the obtained four-dimensional effective theory (4) supplied by the 55 component of five-dimensional Einstein equation. For cosmological applications, the five-dimensional metric is chosen to be a spatial flat in the three spatial dimensions parallel to the brane with an extra dimension transversal to the time for simplicity. This corresponds to the ansatz

$$ds^2 = -n^2(t,y)dt^2 + a^2(t,y)\delta_{ij}dx^i dx^j + \varepsilon(t,y)dtdy + dy^2, \tag{5}$$

where ε is first-order small scale perturbation. We assume that the energy-momentum tensor on brane is that of a perfect fluid with energy density ρ and pressure p

$$\tau_{00} = n^2\rho, \ \tau_{ij} = a^2\delta_{ij} p.$$

Substituting this stress-energy tensor and metric (5) into junction condition equation (2), we obtain

$$\begin{aligned} \frac{a'}{a}\Big|_{y=0} &= -\frac{1}{6}\rho, \\ \frac{n'}{n}\Big|_{y=0} &= \frac{1}{6}(2\rho + 3p). \end{aligned} \tag{6}$$

Here we fix, a prime stands for the derivative with respect to y, a dot denotes the derivative with respect to t, H_0 is the Hubble parameter, and the temporal gauge by imposing the condition $n|_{y=0} = 1$, i.e. we choose the time what corresponds to the cosmic time in the brane.

Using the junction condition (6), the time component of four-dimensional Einstein equation (4) is

$$\frac{1}{36}\rho^2 + \frac{1}{6}\varepsilon\rho\varepsilon' - \frac{2}{3}\varepsilon\rho H_0 - \frac{1}{3}\varepsilon(2\rho + 3p)H_0 - \frac{1}{3}\varepsilon\dot{\rho} - H_0^2 + H_0\varepsilon' = 0. \tag{7}$$

At low energy, we can assume safely $\rho \ll H_0$ i.e. $\frac{\rho}{H_0}$ is first-order small. Furthermore, noting that ε is also first-order small, (7) can be simplified

$$-\frac{1}{3}\varepsilon\dot{\rho} - H_0^2 + H_0\varepsilon' = O(\rho^2), \tag{8}$$

where all higher-order small terms are denoted as $O(\rho^2)$. Consider the cross component of five-dimensional Einstein equation on brane

$$\dot{\rho} + 3H_0(\rho + p) + \varepsilon(6\varepsilon' H_0 - \dot{\varepsilon}\rho - 12H_0^2 - 6\dot{H}_0) = O(\rho^2), \tag{9}$$

with self-consistent condition $\tau_{a5} = \frac{\varepsilon}{2}\rho$. This is the energy conservation equation on the brane, which has been generalized to allow a energy flow $\varepsilon(6\varepsilon' H_0 - \dot{\varepsilon}\rho - 12H_0^2 - 6\dot{H}_0)$ onto the brane. Moreover, it implies that $\varepsilon\dot{\rho}$ is higher-order small in (8), which results

$$\varepsilon' = H_0 \tag{10}$$

on brane at leading order. Using (10), the space component is

$$H_0^2 + \frac{\dot{\varepsilon}}{3}\rho + \dot{H}_0 - \varepsilon\dot{\varepsilon}H_0 = O(\rho^2), \tag{11}$$

and 55 component of five-dimensional Einstein equation on brane is

$$2H_0^2 + \frac{\dot{\varepsilon}}{6}\rho + \dot{H}_0 - \varepsilon\dot{\varepsilon}H_0 = O(\rho^2). \tag{12}$$

Canceling the later two terms in above two equations, the standard Friedmann equation is obtained

$$H_0^2 = \frac{\dot{\varepsilon}}{6}\rho + O(\rho^2), \tag{13}$$

with the identification

$$\frac{\dot{\varepsilon}}{6} = \frac{8\pi G_4}{3}. \tag{14}$$

In other words, the Newton's constant suggests the increased slope of extra dimension. We note here that the result is effective at low energy $\rho \ll H_0$ i.e. $\rho \ll \frac{8\pi G_4}{3}$.

To discuss the evolvement of universe, we cancel the $\dot{\varepsilon}\rho$ in (11) and (12), then

$$\dot{H}_0 - \varepsilon\dot{\varepsilon}H_0 + 3H_0^2 = O(\rho^2). \tag{15}$$

When ε is so small, exactly $\varepsilon \ll \frac{\rho}{H_0}$ (Note $\frac{\rho}{H_0}$ is one-order small, so here ε is higher-order small), that the terms $\varepsilon\dot{\varepsilon}H_0$ in (15) can be omitted, the above equation implies $H_0 = \frac{1}{3t}$. It follows that $a \sim t^{\frac{1}{3}}$, and therefore $H_0^2 \sim \frac{1}{a^6}$. Hence it is a stiff fluid dominated universe for any form of matter on the brane, instead with the radiation-dominated one in Ref. [13]. However, for the ε is not so small, i.e. ε is one-order small as $\frac{\rho}{H_0}$, the case is different. In particular, notice the Friedmann equation (13) with the identification equation (14), (15) is

$$\frac{\ddot{a}}{a} = -\frac{16\pi G_4}{3}(\rho - \varepsilon H_0) + O(\rho^2).$$

One can find that $\frac{16\pi G_4}{3}\varepsilon H_0$ is possible to provide a positive acceleration of the universe when $\varepsilon H_0 > \rho$ at present epoch. It is the self-acceleration without the need for sources of dark energy or cosmological constant, and is independent with the known proposals for modified gravity such as brane world DGP theory [31, 32] and generalized $f(R)$ gravity [33–35]. The concrete behavior of the expansion of this universe will be detailed in next section.

4 A Simple Approximate Solution

For explicitly, let us look for an approximate solution to the five-dimensional equations of motion in whole space-time. Similar to Ref. [11], we set $y \in [-\frac{1}{2}, \frac{1}{2}]$ with another brane located at $y = \frac{1}{2}$, for stability of our 3-brane. One can find that the simplest solution for a and n is

$$\begin{aligned} a &= a_0(t)\left(1 - \frac{1}{6}\rho|y|\right), \\ n &= 1 + \frac{1}{6}(2\rho + 3p)|y|. \end{aligned} \tag{16}$$

Substituting it to five-dimensional Einstein equations, one can find the junction condition (6) is filled, $\widehat{\partial_5 K_{ab}}$ is trivial near our brane, and all equations are satisfied everywhere at leading order if the ε satisfies

$$\varepsilon'|_{y=0} = H_0,$$
$$\dot{\varepsilon}|_{y=0} = 16\pi G_4.$$

The simplest solution can be obtained

$$\varepsilon = \frac{H_0}{A}\sin Ay + 16\pi G_4 t + B. \tag{17}$$

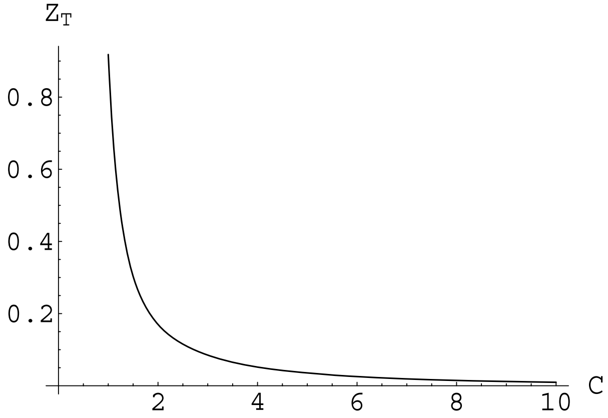

Fig. 1 The transition shift Z_T changes with respect to different parameter C and the present decelerating factor $q(t_0) \simeq -1$. In our theory, Z_T is very small since $C \gg 1$

It is the oscillation function of y and linear function of t on brane, where frequency A and intercept B are constants to tune the order of magnitude of ε to be one-order small. Substituting (17) into (15), the scale factor $a_0(t)$ can be solved at leading order

$$a_0(t) \sim \left[2C + 3\sqrt{2\pi}\,\mathrm{erfi}\left(\frac{\varepsilon_0(t)}{\sqrt{2}}\right)\right]^{1/3}, \tag{18}$$

where $C = 16\pi G_4 D e^{B^2/2}$, D is the integral constant, $\mathrm{erfi}(z)$ is the imaginary error function of z, and $\varepsilon_0 = \varepsilon|_{y=0}$. Thus, we have obtained a solution of five-dimensional Einstein equation in whole space-time.

We further discuss the expansion behavior of this universe. From the scale factor $a_0(t)$ (18), one can get the corresponding decelerating factor q

$$q = 2 - \frac{1}{2}\varepsilon_0 e^{-\varepsilon_0^2/2}\left[2C + 3\sqrt{2\pi}\,\mathrm{erfi}\left(\frac{\varepsilon_0}{\sqrt{2}}\right)\right].$$

Noticing the slope ε_0 is one-order small, the q can be expanded to the first order approximation

$$q = 2 - C\varepsilon_0.$$

Choosing the parameter C satisfied with $\frac{2}{C} < \varepsilon_0(t_0)$ (where t_0 denotes the present time), one can find that this universe may be undergoing an accelerated expansion. To exhibit the complexion of decelerated phase, we evaluate the transition shift Z_T

$$Z_T = \frac{a_0(t_0)}{a_0(t_T)} - 1 \simeq \frac{[2C + 3\sqrt{2\pi}\,\mathrm{erfi}(\frac{2-q(t_0)}{\sqrt{2}C})]^{1/3}}{[2C + 3\sqrt{2\pi}\,\mathrm{erfi}(\frac{2}{\sqrt{2}C})]^{1/3}} - 1,$$

where t_T denotes the time when $q(t_T) = 0$. Using present observation $q(t_0) \simeq -1$, we can plot Z_T for different parameter C in Fig. 1. One can find that when $C \simeq 1$, the transition shift agrees the present observation $Z_T \simeq 0.5$ [25, 26]. Unfortunately, however, the ε_0 is one order small in our theory, which imposes $C \gg 1$ for preserving present acceleration. So the transition shift is very small, meaning that this universe has a decelerated phase in very recent past. Hereto, we can conclude that, though our theory about tiny slope of extra dimension at low energy is so simple that it can not describe the true universe well, we does indeed show that the slope may take role in present acceleration of universe with the decelerated phase in recent past.

5 Summary

In this paper we obtain a novel higher-dimension origin of Newton's constant in brane-world theory and show the expansion behavior of corresponding universe. We first derive the effective four-dimensional Einstein equation at low energy in five-dimensional brane world with a tinily sloping extra dimension, the trivial $\widehat{\partial_5^2 g_{ab}}$, and the static radion field. We find that there still is the similar Israel junction condition at leading order, which results a conventional Newton's constant originated from the cross metrics. Then we derive the standard Friedmann equation at low energy for a spatial flat brane with an extra dimension transversal to the time. It is shown that the Newton's constant suggests the increased slope of extra dimension. When the slope is very small, the corresponding universe is stiff fluid dominated. When the slope increases a little, the theory may provide a self-acceleration of the universe at present epoch. We obtain a simple solution of five-dimensional Einstein equation in whole space-time and illustrate a universe which may accelerate at present epoch and decelerate in the very recent past.

Acknowledgements This work was supported by the National Natural Science Foundation of China under grant No. 10575068, the Natural Science Foundation of Shanghai Municipal Science Technology Commission under grant Nos. 04ZR14059 and 04dz05905 and the CAS Knowledge Innovation Project Nos. KJcx.syw.N2.

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